4.3 23)

$$y=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

$$y'=\frac{(e^x+e^{-x})(e^x+e^{-x})-(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$$

$$=\frac{(e^x)^2+1+1+(e^{-x})^2-\left[(e^x)^2-1-1+(e^{-x})^2\right]}{(e^x+e^{-x})^2}$$

$$=\frac{4}{(e^x+e^{-x})^2}=\left(\frac{2}{e^x+e^{-x}}\right)^2$$

5) $\ln|\tan x|$

$\tan x < 0$

$$\frac{d}{dx}\ln(-\tan x)$$

$$= \frac{1}{-\tan x}\left(-\sec^2 x\right)$$

$$= \frac{\sec^2 x}{\tan x}$$

$\tan x > 0$

$$\frac{d}{dx}\left(\ln(\tan x)\right)$$

$$\frac{1}{\tan x}\left(\sec^2 x\right)$$

$$= \left(\frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos^2 x}\right)$$

$$= \frac{1}{\sin x \cos x}$$

$$\frac{d}{dx}\left(\ln|x|\right) = \frac{1}{x}$$

x neg

$$\frac{d}{dx}\left(\ln(-x)\right)$$

$$\frac{1}{(-x)}\cdot(-1)$$

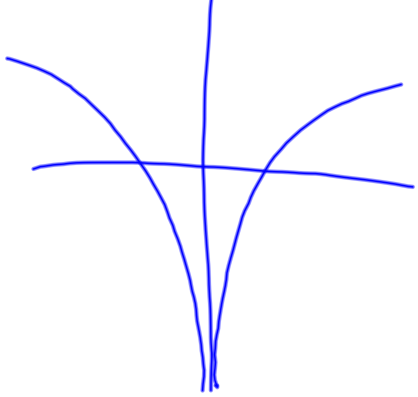
$$= \frac{1}{x}$$

x pos

$$\frac{d}{dx}\left(\ln(x)\right)$$

$$= \frac{1}{x}$$

$\ln|x|$



$$\frac{d}{dx}\left(1 + \log_{10} x\right)$$

$$= 0 + \frac{1}{x \ln 10}$$

"rule"

$$\frac{d}{dx}\left(\log_b x\right) = \frac{1}{x \ln b}$$

13 $\frac{d}{dx}\left(\cos(\ln x)\right)$

$= -\sin(\ln x)\cdot\left(\frac{1}{x}\right)$

9) $y = \ln|x^3 - 7x^2 - 3|$ $\quad \frac{d}{dx}(\ln|x|) = \frac{1}{x}$

$= \frac{1}{x^3-7x^2-3}(3x^2-14x)$

4.3/11

$$y = \sqrt{\ln x} = (\ln x)^{1/2}$$

$$y' = \frac{1}{2}(\ln x)^{-\frac{1}{2}}\left(\frac{1}{x}\right)$$

$$= \frac{1}{2x\sqrt{\ln x}}$$

4.3/29)

$$y = \ln(1 - xe^{-x})$$

$$y' = \frac{1}{1 - xe^{-x}}\left((-1)(e^{-x}) + (-x)(-e^{-x})\right)$$

$$= \underbrace{\frac{xe^{-x} - e^{-x}}{1 - xe^{-x}}}$$

Factor $xe^{-x} - e^{-x}$

$$e^{-x}(x - 1)$$

2.7.4.3)

$$y = e^{(x - e^{3x})}$$

$$y' = e^{(x-e^{3x})} \cdot \left(1 - 3e^{3x}\right)$$

$$y = e^{(x - e^{3x})}$$

$$y = e^{(x - e^{3x})}$$

$$y' = e^{(x - e^{3x})} \cdot \frac{d}{dx}\left(x - e^{3x}\right)$$

$$\frac{d}{dx}\left(x - e^{3x}\right) = 1 - e^{3x} \cdot \frac{d}{dx}(3x)$$

$$= \left(1 - e^{3x} \cdot 3\right)$$

$$\frac{d}{dx}\ln\left(x^3-\frac{1}{x^2}\right) \qquad -x^{-2}$$

$$=\frac{1}{x^3-\frac{1}{x^2}}\left(3x^2+2x^{-3}\right)$$

4.3/16) $y = x\left[\log_2(x^2-2x)\right]^3$

$$y' = (1)\left(\left[\log_2(x^2-2x)\right]^3\right) + (x)\left(\frac{d}{dx}\left[\log_2(x^2-2x)\right]^3\right)$$

$$\frac{d}{dx}\left[\log_2(x^2-2x)\right]^3 = 3\left[\log_2(x^2-2x)\right]^2 \cdot \frac{d}{dx}\log_2(x^2-2x)$$

$$\frac{d}{dx}\left(\log_2(x^2-2x)\right) = \frac{1}{(\ln 2)(x^2-2x)} \cdot (2x-2)$$

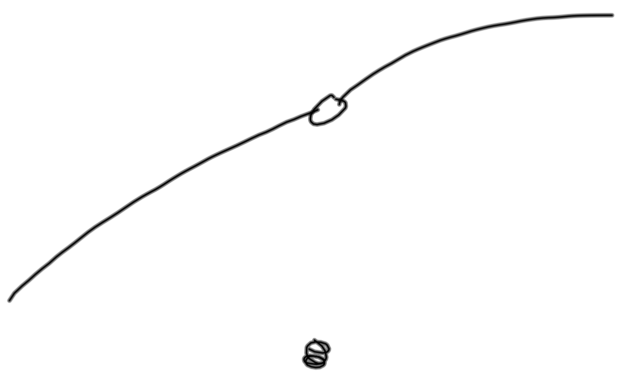
$$y' = \left[\log_2(x^2-2x)\right]^3 + x\left(3\left[\log_2(x^2-2x)\right]^2 \cdot \left(\frac{2x-2}{\ln 2(x^2-2x)}\right)\right)$$

4.3 17 $\frac{d}{dx} \ln\left(\frac{x}{1+x^2}\right)$

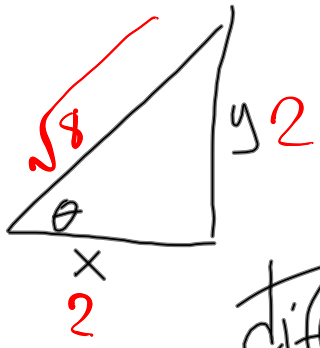
$$= \frac{1}{\frac{x}{1+x^2}} \left(\frac{(1)(1+x^2)-(x)(2x)}{(1+x^2)^2} \right)$$

$$= \frac{1+x^2}{x} \left(\frac{1-x^2}{(1+x^2)^2} \right) = \frac{1-x^2}{x(1+x^2)}$$

limit exists @ c, but f^n isnt cont at c.



3.7/9)



$\cos\theta = \frac{2}{\sqrt{8}}$

$\cos^2\theta = \frac{4}{8} = \frac{1}{2}$

$\frac{1}{\cos^2\theta} = \frac{1}{1/2} = 2$

$\tan\theta = \frac{y}{x}$

diff wrt t

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{\frac{dy}{dt}\cdot x - y\,\frac{dx}{dt}}{(x)^2}$$

At a certain instant . . .

$x = 2$; $\frac{dx}{dt} = 1$; $y = 2$; $\frac{dy}{dt} = -\frac{1}{4}$

$$\sec^2\theta \, \frac{d\theta}{dt} = \frac{\left(-\frac{1}{4}\right)(2) - (2)(1)}{4}$$

2

$$\frac{d\theta}{dt} = \frac{-\frac{1}{2} - 2}{8} = \frac{-\frac{5}{2}}{8} = -\frac{5}{16}\ \frac{\text{unit}}{\text{sec}}$$

13) $y = \cos(\ln x)$

$$y' = -\sin(\ln x) \cdot \frac{1}{x}$$

4.3/23

$$y = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\frac{d}{dx}\left(e^{-x}\right) = e^{-x}(-1)$$

$$y' = \frac{\left(e^{x}+e^{-x}\right)\left(e^{x}+e^{-x}\right) - \left(e^{x}-e^{-x}\right)\left(e^{x}-e^{-x}\right)}{\left(e^{x}+e^{-x}\right)^2}$$

$$= \frac{\left(e^{x}\right)^2 + 1 + 1 + \left(e^{-x}\right)^2 - \left[\left(e^{x}\right)^2 - 1 - 1 + \left(e^{-x}\right)^2\right]}{\left(e^{x}+e^{-x}\right)^2}$$

15)

$$y = x^3 \log_2(3-2x)$$

$$y' = (3x^2)(\log_2(3-2x)) + (x^3)\left(\frac{-2}{(3-2x)\ln 2}\right)$$

$$\frac{d}{dx}\log_2(3-2x) =$$

$$\frac{1}{(3-2x)(\ln 2)} \cdot (-2)$$

$$\frac{d}{dx}\log_b x = \frac{1}{x \ln b}$$

$$\log_b x = \frac{\ln x}{\ln b}$$

25)

$$y = e^{x\tan x}$$

$$y' = e^{(x\tan x)} \cdot \frac{d}{dx}(x\tan x)$$

$$y' = e^{x\tan x}\left((1)(\tan x) + (x)(\sec^2 x)\right)$$

29)

$$y = \ln(1 - xe^{-x})$$

$$y' = \frac{1}{(1 - xe^{-x})}\left((-1)(e^{-x}) + (-x)(-e^{-x})\right)$$

3.2) 27a)

$$6 = \lim_{x_1 \to 3} \frac{x_1^2 - 9}{x_1 - 3} = f'(3)$$

$$f'(x_0) = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$x_0 = 3$

$f(x_1) = x_1^2$

$f(x) = x^2$

$f(x_0) = 9$

original fn : x^2

$$\frac{d}{dx}(x^2) = 2x$$

$$2x\Big|_{x=3} = 6$$

$$\lim_{x_1 \to 3} \frac{x_1^2 - 9}{x_1 - 3} = \lim_{x_1 \to 3} \frac{(x_1 - 3)(x_1 + 3)}{x_1 - 3}$$

$$= \lim_{x_1 \to 3} x_1 + 3 = 6$$

3.2/27b

$$f'(1) = \lim_{\Delta x \to 0} \frac{\sqrt{1+\Delta x} - 1}{\Delta x}$$

when $f(x) = \sqrt{x}$

$$f(x+\Delta x) = \sqrt{1+\Delta x}$$

$$f(x) = \sqrt{x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{x+\Delta x - x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

4 def$^{\underline{n}}$ of derivative

$$f'(x) = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$